

The CP-Violating Asymmetry in $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ *

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Abstract

We update the theoretical analysis of the CP-violating asymmetry in the decay $K_L \rightarrow \pi^+ \pi^- e^+ e^-$, relying on chiral perturbation theory and on the most recent phenomenological information. With the experimentally determined magnetic amplitude and branching ratio as input, the asymmetry can be calculated with good accuracy. The theoretical interpretation of the sign of the asymmetry is discussed.

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1. A large CP-violating asymmetry in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ was originally predicted by Sehgal and Wanninger [1]. The effect is almost entirely due to indirect CP violation in $K^0 - \bar{K}^0$ mixing. The predicted asymmetry in the angle between the $\pi^+\pi^-$ and the e^+e^- planes has been confirmed experimentally [2, 3]. The purpose of this note is to update the theoretical analysis of the asymmetry [1, 4, 5, 6, 7, 8] using the most recent phenomenological input available and employing the methods of chiral perturbation theory. The latter will mainly be invoked to estimate higher-order corrections but also to interpret the observed sign of the asymmetry.

2. The amplitude for the decay $K_L(p) \rightarrow \pi^+(p_1)\pi^-(p_2)e^+(k_+)e^-(k_-)$ is expressed in terms of three invariant form factors E_1, E_2 (electric) and M (magnetic):

$$\begin{aligned} A(K_L \rightarrow \pi^+\pi^-e^+e^-) &= \frac{e}{q^2} \bar{u}(k_-)\gamma^\mu v(k_+) V_\mu \\ V_\mu &= iE_1 p_{1\mu} + iE_2 p_{2\mu} + M\varepsilon_{\mu\nu\rho\sigma} p_1^\nu p_2^\rho q^\sigma \\ q &= k_+ + k_- . \end{aligned} \quad (1)$$

We use the set of variables originally introduced by Cabibbo and Maksymowicz [9] for K_{e4} decays. With this choice, the form factors E_1, E_2, M depend on s_π (invariant mass squared of the two pions), q^2 (invariant mass squared of the leptons) and θ_π (angle of the π^+ in the $\pi^+\pi^-$ center-of-mass system with respect to the dipion line of flight in the kaon rest frame). The two remaining Dalitz variables are θ_l , the corresponding angle for the positron, and Φ , the angle between the dipion and dilepton planes in the kaon rest frame.

After integration over four of these variables, the differential decay rate with respect to Φ assumes the general form¹ [10, 11]

$$\frac{d\Gamma}{d\Phi} = I_1 + I_2 \cos \Phi + I_3 \sin \Phi + I_4 \cos 2\Phi + I_5 \sin 2\Phi . \quad (2)$$

Under a CP transformation

$$\begin{aligned} \cos \Phi &\longrightarrow \cos \Phi \\ \sin \Phi &\longrightarrow -\sin \Phi \end{aligned} \quad (3)$$

so that non-zero I_3, I_5 signify CP violation. It turns out that I_3 is very small in the standard model, being sensitive to direct CP violation only [4]. The quantity of interest here is I_5 that is almost exclusively due to indirect CP violation [4]. A convenient measure of this term is the asymmetry

$$\mathcal{A}_{CP} = \langle \text{sgn}(\sin \Phi \cos \Phi) \rangle = \frac{4I_5}{\Gamma(K_L \rightarrow \pi^+\pi^-e^+e^-)} . \quad (4)$$

¹We use the conventions of Ref. [11], in particular $\varepsilon_{0123} = 1$ and the metric $(+ - - -)$.

In terms of the invariant form factors defined in (1), the asymmetry is given by [10, 11]

$$\mathcal{A}_{CP} = \frac{e^2}{128\pi^8 M_{K^0} \Gamma(K_L \rightarrow \pi^+ \pi^- e^+ e^-)} \int d_{LIPS} \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu k_+^\rho k_-^\sigma \frac{(p_1 - p_2) \cdot (k_+ - k_-)}{q^4} \text{sgn}(\sin \Phi \cos \Phi) \text{Im}[(E_1 - E_2)M^*] \quad (5)$$

where d_{LIPS} is the invariant phase space integration measure. We will also consider the asymmetry for certain cuts (in q^2). In this case, both the phase space integration in the numerator of (5) and $\Gamma(K_L \rightarrow \pi^+ \pi^- e^+ e^-)$ in the denominator must be modified accordingly.

When CP is conserved we have $E_2(p_1, p_2, q) = E_1(p_2, p_1, q)$. Therefore, only CP-violating contributions to E_1, E_2 matter for the numerator in (5).

The most recent published result for the asymmetry comes from the KTeV-Collaboration [2]:

$$\mathcal{A}_{CP} = 13.6 \pm 2.5 \text{ (stat)} \pm 1.2 \text{ (syst)} \% . \quad (6)$$

The preliminary result from NA48 [3] is fully compatible with this value.

Before delving into the theoretical analysis, we comment on the observed sign of the asymmetry (6). With our conventions [11] and with the assumption that $\text{Im}[(E_1 - E_2)M^*]$ has a unique sign all over phase space, the theoretical expression (5) implies

$$\text{sgn } \mathcal{A}_{CP} = \text{sgn } \text{Im}[(E_1 - E_2)M^*] . \quad (7)$$

As we shall show in the sequel, $\text{Im}[(E_1 - E_2)M^*]$ does in fact have a unique sign and must therefore be positive according to Eqs. (6) and (7).

3. At lowest order in the low-energy expansion, the electric amplitudes correspond to pure Bremsstrahlung:

$$\begin{aligned} E_1 &= \frac{-2ieA^{\text{tree}}(K_L \rightarrow \pi^+ \pi^-)}{q^2 + 2p_1 \cdot q} = \frac{-2\sqrt{2}e\eta_{+-}A_0^{\text{tree}}}{q^2 + 2p_1 \cdot q} \\ E_2 &= -E_1(p_1 \rightarrow p_2) . \end{aligned} \quad (8)$$

From the dominant $\Delta I = 1/2$ weak Hamiltonian, the $I = 0$ amplitude A_0^{tree} for $K_S \rightarrow \pi\pi$ decays is given by

$$A_0^{\text{tree}} = \sqrt{2}G_8 F(M_K^2 - M_\pi^2) , \quad (9)$$

with $G_8 = 9.1 \cdot 10^{-6} \text{ GeV}^{-2}$ and $F = F_\pi = 92.4 \text{ MeV}$ at tree level. The current values for the CP-violating quantity η_{+-} are [12]

$$\begin{aligned} |\eta_{+-}| &= (2.276 \pm 0.017) \cdot 10^{-3} \\ \Phi_{+-} &= \arg \eta_{+-} = (43.3 \pm 0.5)^\circ . \end{aligned} \quad (10)$$

The magnetic tree-level amplitude starts at $O(p^4)$ where it is completely given in terms of low-energy constants of the nonleptonic weak Lagrangian of $O(p^4)$ [13, 14]. In the notation of Ref. [15], the leading-order magnetic amplitude is written as

$$M = -\frac{eG_8}{2\pi^2 F}(a_2 + 2a_4) \quad (11)$$

with dimensionless coefficients a_2, a_4 of order one. As shown in Ref. [15], the chiral anomaly induces positive contributions to these coefficients. If the anomaly-induced contributions were the dominant ones, we would expect $a_2 + 2a_4$ to be positive.

It is easy to check for the tree-level amplitude that the CP-violating quantity $\text{Im}[(E_1 - E_2)M^*]$ has a definite sign that equals the sign of $a_2 + 2a_4$. From (6) and (7) we would therefore conclude that $a_2 + 2a_4$ is positive supporting the hypothesis that the contributions from the chiral anomaly dominate the magnetic amplitude (11).

However, this cannot be the whole story. Assuming a constant magnetic amplitude, one can extract this amplitude from the branching ratio for $K_L \rightarrow \pi^+\pi^-\gamma$. Following Sehgal and Wanninger [1] (the same procedure is used in Refs. [4, 5]), one obtains in their notation

$$M = -\frac{0.76e|f_s|}{M_K^4}, \quad (12)$$

with $|f_s| = 3.9 \cdot 10^{-7}$ GeV denoting the absolute value of the $K_S \rightarrow \pi^+\pi^-$ amplitude. The resulting asymmetry at tree level is

$$\mathcal{A}_{CP}^{\text{tree}} = 7.7 \% , \quad (13)$$

much smaller than the experimental result (6). Clearly, higher-order corrections are necessary to understand the asymmetry.

4. Chiral corrections to the tree-level amplitudes were already considered in Ref. [6]. Those authors included one-loop corrections to the absorptive parts of both electric and magnetic form factors. Whereas the corrections are small for the magnetic amplitude they are sizable for the electric form factors, mainly due to the large final state interactions of two pions with $I = 0$ in an S -wave, as first pointed out by Sehgal and Wanninger [1]. To some extent, Elwood et al. [6] compensated the neglect of dispersive contributions by using the tree-level value of the weak coupling constant G_8 also in the loop amplitude. On the other hand, this is partly double counting because G_8 should be reduced by about 30 % [16, 17] when effects of $O(p^4)$ are included in the amplitude for $K_1^0 \rightarrow \pi^+\pi^-$.

We propose here a different procedure for the CP-violating electric amplitude. We first decompose the electric form factors into a Bremsstrahlung part and a direct-emission piece:

$$E_i = E_i^{\text{B}} + E_i^{\text{DE}} \quad (i = 1, 2) . \quad (14)$$

In the first part, we use the phenomenological $K_S \rightarrow \pi^+\pi^-$ amplitude,

$$\begin{aligned} E_1^{\text{B}} &= -\frac{2\sqrt{2}e\eta_{+-}}{q^2 + 2p_1 \cdot q} \left[A_0 e^{i\delta_0^0} + \frac{1}{\sqrt{2}} A_2 e^{i\delta_0^2} \right] \\ E_2^{\text{B}} &= -E_1^{\text{B}}(p_1 \rightarrow p_2), \end{aligned} \quad (15)$$

with isospin amplitudes $A_0 = 2.71 \cdot 10^{-7}$ GeV, $A_2 = 0.12 \cdot 10^{-7}$ GeV taken from experiment. The pion-pion phase shifts δ_0^0, δ_0^2 at $s = M_K^2$ are taken from a new analysis of pion-pion scattering combining dispersion theory (Roy equations) with chiral perturbation theory [18, 19]:

$$\delta_0^0 = 39.1^\circ, \quad \delta_0^2 = -8.5^\circ. \quad (16)$$

For the CP-violating direct-emission part, we have calculated the amplitude of $O(p^4)$ for $K_1^0 \rightarrow \pi^+ \pi^- \gamma^*$, including both a local amplitude derived from the nonleptonic weak Lagrangian of $O(p^4)$ [13, 14] and a loop amplitude restricted to the dominant pion loops. Of course, one has to project out the Bremsstrahlung part of $O(p^4)$ that is already included in E_i^B in (15). Altogether, we have

$$\begin{aligned} E_1^{\text{DE}} &= E_1^{\pi\text{-loops}}(\mu = M_\rho) \\ &\quad + \frac{2e\eta_{+-}G_8}{3F} q^2 [N_{14}^r(M_\rho) - N_{15}^r(M_\rho) - 3(N_{16}^r(M_\rho) + N_{17})] \\ &\quad + \frac{4e\eta_{+-}G_8}{F} p_2 \cdot q [N_{14}^r(M_\rho) - N_{15}^r(M_\rho) - N_{16}^r(M_\rho) - N_{17}] \\ E_2^{\text{DE}} &= -E_1^{\text{DE}}(p_1 \leftrightarrow p_2). \end{aligned} \quad (17)$$

The explicit form of the direct-emission pion-loop amplitude $E_1^{\pi\text{-loops}}(\mu = M_\rho)$ can be found in Ref. [20]. Because we have only included the dominant pion loops there is a residual, but numerically unimportant scale dependence in (17). We have chosen the usual renormalization scale $\mu = M_\rho$. The numerical values for the low-energy constants $N_i^r(M_\rho)$ (N_{17} is scale independent) are taken from a recent analysis [20, 21] of the branching ratio $B(K_L \rightarrow \pi^+ \pi^- e^+ e^-)$. Anticipating the numerical discussion, the direct-emission form factors E_i^{DE} are negligible for \mathcal{A}_{CP} , especially when integrated over the whole phase space.

For the magnetic amplitude, we must be less ambitious for the time being. Already at $O(p^4)$, we cannot claim to be able to calculate the coefficients in Eq. (11). Moreover, $\eta - \eta'$ mixing is known to produce a big contribution of $O(p^6)$ [22] that interferes destructively with the amplitude induced by the chiral anomaly. In addition, there are two apparently non-equivalent versions of implementing vector and axial-vector exchange in nonleptonic weak transitions [23, 24].

On the other hand, there exists strong experimental evidence for higher-order effects parametrized in terms of a ρ -dominated form factor [2, 25]. Including final state interactions appropriate for P -wave $\pi\pi$ scattering, we therefore adopt the magnetic amplitude measured by KTeV,

$$M = \frac{e|f_S|}{M_K^4} \tilde{g}_{M1} e^{i\delta_1^1(s_\pi)} \left[1 + \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2M_K E_\gamma^*} \right], \quad (18)$$

with E_γ^* the total lepton energy in the kaon rest frame and with² [2]

$$\tilde{g}_{M1} = 1.35 \begin{smallmatrix} +0.20 \\ -0.17 \end{smallmatrix}, \quad a_1/a_2 = (-0.720 \pm 0.028) \text{ GeV}^2. \quad (19)$$

²The quantity a_2 in (18) must not be confused with the same expression in (11).

The $I = J = 1$ phase shift is parametrized as

$$\delta_1^1(s_\pi) = \frac{1}{2} \arcsin \left[\frac{4q_\pi^3}{\sqrt{s_\pi}} (a_1^1 + b_1^1 q_\pi^2 + c_1^1 q_\pi^4) \right] \quad (20)$$

with

$$\begin{aligned} a_1^1 &= 0.038/M_{\pi^+}^2, \quad b_1^1 = 0.0057/M_{\pi^+}^4, \quad c_1^1 = 0.001/M_{\pi^+}^6, \\ q_\pi &= (s_\pi/4 - M_{\pi^+}^2)^{1/2}. \end{aligned} \quad (21)$$

The slope parameters a_1^1, b_1^1 are taken from Refs. [18, 19] and the coefficient c_1^1 is introduced to reproduce the correct phase shift at $s = M_K^2$.

With the sign convention of (15), the magnetic coupling \tilde{g}_{M1} must be positive in order to reproduce the measured sign of the asymmetry. Unfortunately, there is at present no unique way to infer from (18) and the experimental values (19) the sign of the lowest-order combination $a_2 + 2a_4$ in (11). A recent analysis of $K \rightarrow \pi\pi\gamma$ transitions by D'Ambrosio and Gao [26] in the framework of the vector-field representation for (axial-)vector resonances finds that both signs are possible depending on the coupling strength of spin-1 exchange. A deeper understanding of higher-order effects in nonleptonic weak interactions will be necessary before one could claim that the chiral anomaly is largely responsible for the measured sign of the asymmetry.

5. With electric form factors given in Eqs. (15) and (17) and with the magnetic amplitude (18) we obtain for the total integrated asymmetry

$$\mathcal{A}_{CP} = 13.7 \% \quad (22)$$

if all input quantities are taken at their respective mean values.

This value agrees with previous theoretical estimates and with the experimental results from KTeV and NA48. The main issue we want to address here is the theoretical uncertainty of this prediction. Electric and magnetic amplitudes are on quite a different footing in this respect. Except for the P -wave phase shift $\delta_1^1(s_\pi)$ that we will lump together with the phases occurring in E_i^B , we cannot ascribe a meaningful theoretical error to the magnetic amplitude. We will therefore scale the prediction (22) to the measured magnetic coupling \tilde{g}_{M1} and include the experimental error of the ratio a_1/a_2 explicitly. Accounting for the measured branching ratio in the same way, we arrive at

$$\mathcal{A}_{CP} = \frac{3.63 \cdot 10^{-7}}{B(K_L \rightarrow \pi^+\pi^-e^+e^-)} \cdot \frac{\tilde{g}_{M1}}{1.35} (13.7 \pm 1.3) \% \quad (23)$$

where the given error is due to the error of a_1/a_2 in (19) only. Future experimental improvements and correlations between the measured values for $B(K_L \rightarrow \pi^+\pi^-e^+e^-)$, \tilde{g}_{M1} and a_1/a_2 can easily be incorporated in this formula.

In contrast, the uncertainty of \mathcal{A}_{CP} related to the electric form factors is fully under control theoretically. For the total asymmetry under consideration here, the CP-violating direct-emission amplitude (17) is completely negligible and does not affect \mathcal{A}_{CP} to the

accuracy given. Of course, this is a consequence of the dominance of the Bremsstrahlung form factors (15) at small q^2 .

Inclusion of the $I = 2$ amplitude A_2 is itself a small effect contributing 0.25 % to \mathcal{A}_{CP} . The relatively big errors of A_2 and δ_0^2 due to isospin violation [27] and electromagnetic corrections [28] have therefore no impact on \mathcal{A}_{CP} . On the other hand, those corrections are small for A_0 and δ_0^0 that do matter for the asymmetry. Assigning an uncertainty $\pm 0.05 \cdot 10^{-7}$ GeV to A_0 induces $\Delta \mathcal{A}_{CP} = 0.25$ %. Likewise, the error for $|\eta_{+-}|$ given in (10) propagates into $\Delta \mathcal{A}_{CP} = 0.10$ %.

Because of the dominance of Bremsstrahlung, the phases relevant for \mathcal{A}_{CP} enter effectively in the combination $\delta_0^0 - \delta_1^1(s_\pi) + \Phi_{+-}$. From the recent analysis of pion-pion phase shifts [18, 19] we extract an uncertainty of 0.6° for $\delta_0^0 - \delta_1^1(s_\pi)$. Together with the error of Φ_{+-} displayed in (10), the conservative uncertainty $\Delta(\delta_0^0 - \delta_1^1(s_\pi) + \Phi_{+-}) = 1^\circ$ gives rise to $\Delta \mathcal{A}_{CP} = 0.05$ %.

Combining the uncertainties from A_0 , $|\eta_{+-}|$ and from the phases in quadrature, we add the resulting theoretical error of 0.3 % to the previous one due to a_1/a_2 to obtain our final result for the total asymmetry:

$$\mathcal{A}_{CP} = \frac{3.63 \cdot 10^{-7}}{B(K_L \rightarrow \pi^+ \pi^- e^+ e^-)} \cdot \frac{\tilde{g}_{M1}}{1.35} (13.7 \pm 1.3 \pm 0.3) \% . \quad (24)$$

Given the magnetic amplitude and the branching ratio $B(K_L \rightarrow \pi^+ \pi^- e^+ e^-)$, the CP-violating asymmetry can be calculated to a relative accuracy of about 2 %.

6. Following the authors of Refs. [5, 6, 7], we consider the asymmetry also for different cuts in q^2 , the invariant mass squared of the leptons. For this purpose, the chiral amplitude to $O(p^4)$ [5, 21] is used to calculate the rate $\Gamma(K_L \rightarrow \pi^+ \pi^- e^+ e^-)$ in the denominator of the asymmetry for the cuts in question. As already mentioned, the low-energy constants N_i^r are taken from a recent analysis [20, 21] of the total branching ratio $B(K_L \rightarrow \pi^+ \pi^- e^+ e^-)$.

Suppressing errors, the results are collected in Table 1. One immediate conclusion is that the asymmetry is maximal for $q^2 \gtrsim (10 \text{ MeV})^2$.

Compared to the previous situation for the total asymmetry with $q^2 \geq 4m_e^2$, there are now additional uncertainties that increase with the lower limit on q^2 . In the numerator of the asymmetry, the direct-emission form factors E_i^{DE} in (17) become more important in comparison with the Bremsstrahlung amplitude. Likewise, the rate in the denominator becomes more sensitive to the inaccurately known weak low-energy constants N_i^r as one moves to larger q^2 .

For large q^2 , both the asymmetry and the rate decrease rapidly. For illustration, we therefore choose a realistic cut of $q^2 > (40 \text{ MeV})^2$. The CP-conserving amplitude of $O(p^4)$ depends on the combination $Z(\mu) = N_{14}^r(\mu) - N_{15}^r(\mu) - 3(N_{16}^r(\mu) - N_{17})$ of weak low-energy constants. With the mean value $Z(M_\rho) = 0.023$ extracted from the total branching ratio [21], about 13 % of all events (almost 16 % of those with $q^2 > (2 \text{ MeV})^2$ as in the KTeV experiment [2]) satisfy $q^2 > (40 \text{ MeV})^2$.

However, these numbers are rather sensitive to the precise value of $Z(M_\rho)$. In Table 2, the dependence of the rate on the electric amplitude is exhibited for $q^2 > (40 \text{ MeV})^2$.

Table 1: The asymmetry \mathcal{A}_{CP} for different cuts in q^2 , the invariant mass squared of the lepton pair.

$q^2 > [\text{MeV}^2]$	\mathcal{A}_{CP} in %
entire phase space	13.7
2^2	15.2
10^2	15.7
20^2	14.0
30^2	12.1
40^2	10.2
60^2	7.2
80^2	4.8
100^2	3.2
120^2	2.0
180^2	0.4

About 25% of the rate is due to $Z(M_\rho)$ being different from zero. This also documents that $Z(M_\rho)$ can in principle be extracted with much higher precision from the partial rate than from the total branching ratio. In the sample accumulated by KTeV [29], several hundred events are expected to satisfy $q^2 > (40 \text{ MeV})^2$.

As already mentioned, the direct-emission amplitude of $O(p^4)$ also enters the numerator of the asymmetry. In contrast to the rate where the CP-conserving electric amplitude of $O(p^4)$ overtakes the CP-violating Bremsstrahlung amplitude (8) of $O(p^2)$ already at fairly small q^2 [21], only the CP-violating part of direct emission matters in the numerator. For the considered cut $q^2 > (40 \text{ MeV})^2$ this is still only a small correction to the leading-order Bremsstrahlung. Numerically, the correction is smaller than the uncertainty due to A_0 and it will be included in the theoretical error.

Consequently, for reasonable cuts in q^2 the asymmetry can still be predicted rather precisely in terms of the magnetic amplitude and the branching ratio. For $q^2 > (40 \text{ MeV})^2$, we obtain

$$\mathcal{A}_{CP} = \frac{4.75 \cdot 10^{-8}}{B(K_L \rightarrow \pi^+\pi^-e^+e^-; q^2 > (40 \text{ MeV})^2)} \cdot \frac{\tilde{g}_{M1}}{1.35} (10.2 \pm 1.1 \pm 0.3) \% \quad (25)$$

where the first error is again due to the error of a_1/a_2 in (19), the second one being the genuine theoretical error.

7. The electric direct-emission amplitude for the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ has been calculated in chiral perturbation theory. The CP-conserving part is important for the rate because it dominates the CP-violating Bremsstrahlung amplitude already for rather low invariant masses of the lepton pair. With appropriate cuts in q^2 , it will be possible to

Table 2: Dependence of the rate $\Gamma_{\text{cut}}(K_L \rightarrow \pi^+\pi^-e^+e^-)$ on the electric amplitude [21] for $q^2 > (40 \text{ MeV})^2$. $E_i^{O(p^4)}$ denotes the complete electric amplitude to $O(p^4)$ that depends on the combination $Z(\mu) = N_{14}^r(\mu) - N_{15}^r(\mu) - 3(N_{16}^r(\mu) - N_{17})$ of weak low-energy constants. The magnetic amplitude is given in (18).

	$\Gamma_{\text{cut}}/\Gamma_{\text{cut}}(O(p^4))$
E_i^B	0.60
$E_i^{O(p^4)}$ with $Z(M_\rho) = 0.023$	1.
$E_i^{O(p^4)}$ with $Z(M_\rho) = 0$	0.75

extract the relevant combination of low-energy constants in the CP-conserving amplitude of $O(p^4)$ with good precision.

In contrast, the CP-violating asymmetry is quite insensitive to electric direct emission. This allows for an accurate calculation of the asymmetry once the magnetic amplitude and branching ratio have been determined experimentally. For realistic cuts in q^2 , the asymmetry can be predicted with a relative accuracy of $2 \div 3\%$.

The contribution of the chiral anomaly to the magnetic amplitude of $O(p^4)$ leads to the observed sign of the asymmetry. However, higher-order terms in the magnetic amplitude, clearly required by experiment but not reliably calculable in chiral perturbation theory, make this connection less conclusive.

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